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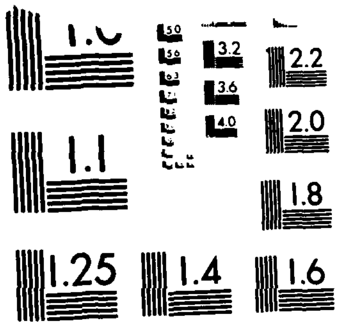

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This report summarizes the activities in support of the Air Force Research Project AFOSR-83-0255 during the past year. Efforts have been made to develop, test and analyze new fast techniques in matrix analysis for structural computations and least squares problems. Applications of this work include structural design and dynamics, and least squares filtering in signal processing. Implementations and tests have been made on modern high performance architectures such as the Cray X-MP, Alliant FX/8, Sequent Balance and the Intel iPSC Hypercube. Our recent work on parallel algorithms for near real-time signal processing computations has led to especially significant results.

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Fast Algorithms for Structural Optimization and Least Squares

R. J. Plemmons

August 1987

I. Project Objectives

Researchers in scientific computation recognize that to achieve the speeds necessary to solve the new complex scientific and engineering problems of significant impact to the DOD communtiy, requires radical reorganization of traditional algorithms in matrix analysis. It is not sufficient to just implement old algorithms in a parallel processing environment. New fast algorithms for the modern generation of supercomputers (such as the Cray X-MP) and mini-supercomputer systems (such as the Alliant FX/8) as well as new massively parallel multiprocessors, are essential. In order to meet the challanges of this emerging new generation of machines, it is the goal of this project to develop techniques in matrix computations for efficient implementation on advanced architectures. Significantly, applications of our work to the practical real-world problems of structural optimization and least squares estimation methods in signal processing are being made.

In the area of **structural optimization** we are concerned with the fundamental problem of elastic analysis - that of finding the stresses and strains and solving optimal redesign problems, given a finite element model of a complex structure and a set of external loads. To obtain the solution of this constrained minimization problem, a variety of algorithms involving the displacement method or the force method can be applied. While the advantages of implementing one of these methods over the other on serial computers have been widely studied, the effects of parallelism in performing the matrix computations have not received a great deal of attention until recently. Our work on this topic thus far has led to publications in **Numerische Mathematik**, the **SIAM Journal on Algebraic and Discrete Methods**, the **SIAM Journal on Scientific and Statistical Computing**, and in **Computer Methods in Applied Mechanics and Engineering.** Our goals here continue to be the development and testing of complete finite element structural optimization packages on machines such as the Cray X-MP and the Alliant FX/8, and their comparison with traditional serial methods in packages such as NASTRAN.

Our main objective in **least squares computations** this year has been to complete the error analysis and testing of new recursive orthogonal and hyperbolic rotation algorithms for signal processing. This is joint work with a Ph.D. student, C. T. Pan, and with S. T. Alexander from the NCSU Department of Electrical and Computer Engineering. Our schemes are amenable to implementation on a variety of vector and parallel processing systems, such as the Alliant FX/8 and the Intel iPSC Hypercube. This work in developing near real-time algorithms has produced some especially significant recent results. *The results we are obtaining here may very likely have a significant impact with DOD researchers who are interested in near real-time computations in, for example, control and signal processing.*

The goals of this research are to investigate the theoretical aspects of the computations as well as to develop new technologies for solving important problems in an efficient and stable way on modern high performance architectures. Here there two areas of particular excitement in our project . We are close to the establishment of a framework for testing and comparing parallel algorithms for structural optimization against the more traditional approaches in commercial software. In addition, we are developing new tools for least squares computations in signal processing necessary to meet the challenges of solving near real-time problems in a stable way on the new generation of multiprocessor systems.

Abstracts of some major findings obtained during the past year of this project are provided in the next section.

II. Abstracts of Major Results

1. **Computational Structural Mechanics on High Performance Architectures:** The major focus of this work is a detailed study of the vectorization and parallelization of new and existing variations of the Displacement and Force Methods in the engineering analysis of large-scale structures. Considering the increasing demands on the structural engineer to analyze larger and more complex structures, the need for vectorization and/or multiprocessing of the numerical schemes is essential. We have used two high performance architectures in this work: an Alliant FX/8 and a Cray X-MP (made available by the NSF at the University of Illinois NCSA). Implementation and performance evaluations for a variety of approaches on these architectures have indicated that an element-by-element preconditioned conjugate gradient scheme produces superior performance. Some of the results of this study were presented at the First World Congress on Computational Mechanics at Austin ,TX. (This work includes a joint project with M. W. Berry at the Illinois CSRD.)

2. **Iterative Methods for Equality Constrained Least Squares Problems:** We consider the linear equality constrained least squares problem (LSE) of minimizing the norm $\|c - Gx\|_2$ subject to the constraint $Ex = p$. A preconditioned conjugate gradient method is applied to the Kuhn- Tucker equations associated with the LSE problem. This method is compared to a block SOR method and is clearly superior to it. We show that the method is well suited for structural optimization problems in reliability analysis and optimal design. Numerical tests on an Alliant FX/8 and a Cray X-MP using some practical structural analysis data exhibit the efficiency of the method. Applications also have been made to filtering methods in signal processing. Here the scheme has the definite advantage that the solution x is easy to update after a rank one modification of the matrix G. (This is joint work with J. Barlow at Penn State University and Nancy Nichols at NCSU.)

3. **A Two-Level Preconditioned Conjugate Gradient Scheme:** The conjugate gradient algorithm is one of the most efficient methods for solving a variety of problems arising in signals, systems and control, and it has been successfully implemented on various vector computers. In part as an effort to efficiently implement this algorithm on parallel processors, a two-level preconditioning scheme is proposed here and tested on an Alliant FX/8 multiprocessor system. The scheme is based on applying the SSOR and incomplete Cholesky preconditioners simultaneously to a partitioned form of the coefficient matrix A. The two-level preconditioner appears to be especially well-suited for the case where A has a bordered block diagonal form commonly arising in domain

decomposition or substructuring type problems. (This is joint work with the Ph.D. student D. J. Pierce who is now with Boeing Computer Services.)

4. **Analysis of a Recursive Least Squares Hyperbolic Rotation Algorithm for Signal Processing**: The application of hyperbolic plane rotations to the least squares downdating problem arising in windowed recursive signal processing is studied. A forward error analysis is given to show that the algorithm can be expected to perform well in the presence of rounding errors, provided the problem is not too ill-conditioned. The hyperbolic rotation algorithm is shown to be forward (weakly) stable and, in fact, comparable to an orthogonal downdating method shown to be backward stable by Stewart. Numerical comparisons are made with Stewart's method as implemented in LINPACK. These tests collaborate our error analysis which indicates that the two methods should result in similar accuracy. However, the hyperbolic scheme under consideration requires $n^2/2$ fewer multiplications for each downdating step, where n is the number of least squares filter coefficients. In addition, it is much more amenable to implementation on a variety of vector and parallel machines. (This is joint work with the Ph.D. student C. T. Pan and with S. T. Alexander from the NCSU Department of Electrical and Computer Engineering.)

5. **Numerical Properties of a Hyperbolic Rotation Method for Windowed RLS Filtering**: Numerical properties of the hyperbolic rotation method for windowed RLS filtering are examined. This matrix oriented approach is important from two standpoints: (1) it provides the LS predictor for a sliding window block of data, and (2) it is amenable to parallel implementation. It is shown how a hyperbolic rotation matrix can be constructed to update the Cholesky factor as a function of the previous Cholesky factor and the data in the sliding window. (This is also joint work with the Ph.D. student C. T. Pan and with S. T. Alexander from the NCSU Department of Electrical and Computer Engineering.)

6. **A Sharp Bound for Products of Hyperbolic Plane Rotations.** An algorithm for downdating a least squares problem using hyperbolic plane rotations has recently been presented and analyzed by Alexander, Pan and Plemmons. Their analysis of the numerical stability of the algorithm rests on the existence of a tight bound on the product of norms of a certain collection of hyperbolic rotations. The main result of this paper establishes the required tight bound by use of combinatoric relationships. (This is joint work between the Ph.D. student C. T. Pan and K. Sigmon of the University of Florida.)

7. **Parallel Algorithms for Least Squares and Related Material**: This report is concerned with the solution of large-scale least squares problems. Special attention is placed on those least squares problems arising in a variety of scientific and engineering problems, including geodetic adjustments and surveys, medical

image analysis, molecular structures, partial differential equations and substructuring techniques in structural engineering. In each of these problems, matrices **A** often arise which possess a block angular structure which reflects the local connection nature of the underlying problem. A new direct-iterative method is proposed which corresponds to a new preconditioner for conjugate gradient type algorithms involving the coefficient matrix **A**. This preconditioner is based on an incomplete hyperbolic Cholesky factorization of the normal equations, without explicit formation of the system. The preconditioner is fully capable of exploiting the block structure of **A**, which arises in the applications mentioned above, resulting in an efficient parallel implementation. (This is the Abstract of the Ph.D. dissertation by D. J. Pierce.)

8. **Hyperbolic Rotations for Downdating the Cholesky Factorization with Applications to Signal Processing:** In many applications the rank one modification , i.e. updating or downdating, the Cholesky factorization of a positive definite matrix **A** is an important computation. There are two standard downdating algorithms: (1) the LINPACK algorithm, which is based on orthogonal Givens rotations, and (2) the hyperbolic rotation algorithm, which is similar to the use of Givens rotations except that hyperbolic functions are used. The LINPACK method is known to be backward stable while the hyperbolic algorithm is faster. This dissertation presents a complete forward error analysis of the hyperbolic rotation algorithm and shows that it is forward (or weakly) stable. A new algorithm for downdating Cholesky factorizations is proposed and tested. This new method is faster than the hyperbolic algorithm and is expected to be as stable as the LINPACK method. Applications of downdating schemes to recursive least squares filtering methods in signal processing are also developed. (This is the Abstract of the Ph.D. dissertation by C. T. Pan.)

III. Abstracts of research in Progress

1. **A New Look at the Force Optimization Method in Structural Mechanics:** The Force Optimization Method of structural analysis was overtaken by the Matrix Displacement Method in the mid-sixties, and has disappeared from the scene except for a few specialized applications, as reported by several authors. The great majority of general purpose finite element programs are now based on the Displacement Stiffness method. The key weakness in the past for the Force Optimization Method on the computer has been the diffilculty of automating matrix optimization schemes in an efficient and stable manner. However, recent developments in the area of fast algorithms for local and global optimization by, e.g., R. Byrd and B. Schnabel, have spurred a revival of interest in the Force Method. Quoting from R. Galligher, "Innovation in the Stiffness Method is reaching a plateau of diminishing return. It is prudent to revisit the Force Method and tap its usefullness". With this in mind, we are beginning a study of fresh new approaches to the optimization based Force Method on modern multiprocessor systems. We are attempting to automate the Force Method for implementation on parallel processors such as the Alliant FX/8 and the Sequent Balance. In this regard, element-by-element preconditioning schemes are leading to some exciting new interest in this approach to computational structural mechanics. (This is joint work with R. E. White.)

2. **Parallel Factorization Schemes for Minimizing a Sum of Euclidean Norms:** The problem of minimizing a weighted sum of Euclidean norms is being considered. Applications include minimal surface computations. A robust parallel algorithm, based on the line-search Newton's Method, is being developed which takes advantage of the structure of the problem in order to fully utilize vectorization and concurrency in the computations. The proposed method can achieve high performance, especially on a machine with an architecture that combines vector and parallel capabilities on a two-level shared memory structure, such as that present on the Alliant FX/8 system. (This represents joint work with S. J. Wright.)

3. **Parallel Rank One Matrix Modifications on Distributed Memory Architectures:** Here we are developing and testing parallel least squares updating and downdating schemes on a 64 node Intel Hypercube. The purpose is to design near real-time algorithms for signal processing applications. The results thus far have been very encouraging. One of the students who will be involved with this project next year has spent the Summer at Oak Ridge developing code on their iPSC64 system. (This is joint work with the student C. Henkel, who is majoring in

Nuclear Engineering at NCSU, and M. T. Heath at the Oak Ridge National Laboratory.)

4. **Fast Algorithms for Updating Least Squares Computations**: The typical parallel bottleneck in updating and downdating least squares computations involving an observation matrix **X** is the solution to triangular systems of equations associated with the Cholesky factor **R** for **X**. (Triangular solvers are inherently serial, although some recent progress has been made in parallel schemes for distributed memory systems.) Our key results in this project involve the development of new parallel algorithms for updating or downdating the inverse matrix $\mathbf{R}^{-1}$, thus avoiding triangular solvers altogether. The results we are obtaining here may very well have a significant impact with DOD researchers interested in near real-time computations in, for example, signal processing.

IV. Project Research Personnel

Robert J. Plemmons - Principal Investigator (2 months)

Daniel J. Pierce - Graduate Research Assistant (1 semester, part time)
Ph.D. May 1987

C. T. Pan - Graduate Research Assistant (2 semesters, part time)
Ph.D. August 1987

V. Technical Publications

1. **Convergence of parallel multisplitting iterative methods,** Linear Algebra and Applications, 88(1987), 39-49. (with M. Neumann)

2. **A conjugate gradient method for the solution of equality constrained least squares problems,** SPIE - Advanced Algorithms and Architectures for Signal Processing, 696(1987), 23-30. (with J. Barlow and N. Nichols)

3. **Parallel multisplitting iterative methods,** Current Trends in Matrix Theory, Ed. by F. Uhlig and R. Grone, North Holland, (1987), 251-254.

4. **Numerical properties of a hyperbolic rotation method for windowed RLS filtering,** Proc. IEEE Conf. on Acoustics, Speach and Signal Processing, (1987), Dallas, TX. (with S. T. Alexander and C. T. Pan)

5. **A sharp bound for products of hyperbolic plane rotations,** SIAM J. on Matrix Analysis and Applications, (1987), to appear. (C. T. Pan and K. Sigmon)

6. **Parallel algorithms for least squares and related material,** Ph.D. dissertation, May 1987, D. J. Pierce. (directed by R. J. Plemmons)

7. **Hyperbolic rotations for downdating the Cholesky factorization with applications to signal processing,** Ph.D dissertation, August 1987, C. T. Pan. (directed by R. J. Plemmons)

8. **Algorithms and experiments for structural mechanics on high performance architectures,** Computer Methods in Applied Mechanics and Engineering, (1987), to appear. (with M. W. Berry)

9. **A two-level preconditioned conjugate gradient scheme,** Proc. of SIAM Conf. on Lin. Alg. in Signals Systems and Control, Ed. by B. Datta and R. J. Plemmons, (1987), to appear. (with D. J. Pierce)

10. **An efficient parallel scheme for minimizing a sum of Euclidean norms,** submitted to the SIAM J. On Scientific and Statistical Computing, (1987), (with S. J. Wright)

VI. Other Activities

1. **Invited Conference Lectures:**

(a). First World Congress on Computational Mechanics, Austin , TX (1986). (with M. Berry)
(b). ICIAM'87 Minisymposium on Linear Algebra in Systems and Control, Paris, France (1987).

2. **Colloquium Lectures:**

(a). Argonne National Laboratory, Argonne, IL (1986).
(b). Air Force Office of Scientific Research, Bolling Air Force Base, DC (1987).
(c). INRIA, Rennes University, Rennes, France (1987).

3. **Conference Organizing Committees:**

(a). SIAM Conference on Lin. Alg. in Signals, Systems and Control, Boston, MA (1987).

(b). First World Congress on Computational Mechanics, Austin, TX (1987).
(c). ICIAM'87 Minisymposium on Linear Algebra in Systems and Control, Paris France, (1987).
(d). Third SIAM Conference on Applied Linear Algebra, Madison, WI, to be held (1988).

4. Editorial and Other Activities:

(a). Elected Member, SIAM Council.
(b). Advisory Editor, Linear Algebra and Applications.
(c). Editorial Board, SIAM J. on Algebraic and Discrete Methods.
(d). Associate Managing Editor, SIAM J. on Matrix Analysis and Applications.

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